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Estimating Aquifer Transmissivity from Specific Capacity Using Matlab

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Abstract

Historically, specific capacity information has been used to calculate aquifer transmissivity when pumping test data are unavailable. This paper presents a simple computer program written in the Matlab programming language that estimates transmissivity from specific capacity data while correcting for aquifer partial penetration and well efficiency. The program graphically plots transmissivity as a function of these factors so that the user can visually estimate their relative importance in a particular application. The program is compatible with any computer operating system running Matlab, including Windows, Macintosh OS, Linux, and Unix. Two simple examples illustrate program usage.

Introduction

A computer technique for estimating transmissivity from specific capacity data is currently available (Bradbury and Rothschild 1985). However, it is written in Basic and does not graphically display results. This paper presents a modified version of the Bradbury-Rothschild iterative solution technique that is written in the Matlab language and listed in the Appendix. A useful new feature includes a 3-D graphical display of results so that the user can quickly estimate the relative importance of aquifer penetration and well efficiency. Potential users should be aware that Matlab must be installed on their computers before the program will function. Alternately, users may convert either the original or revised code to any convenient programming language (e.g., C++, Fortran, Excel, or MathCad). However, Matlab is a powerful tool with numerous capabilities that are not readily found in other languages.

Recall that total drawdown (s_t) observed in a production well can be written (Bouwer 1978) as the sum of drawdown due to formation loss (s_f) and drawdown due to well loss (s_w), or:

$$s_t = s_f + s_w = BQ + CQ^n \tag{1}$$

where B = formation loss coefficient (T/L^2),
C = well loss coefficient (T^2/L^5 if $n = 2$),
Q = well discharge (L^3/T), and
n = an exponent related to wellbore turbulence (typically $1.5 \le n \le 3.5$).

When well efficiency (E) is defined as $E = 100\, s_f/s_t$ and $n = 2$, then C is related to E by:

$$C = \left(\frac{s_t}{Q^2}\right)\left(1 - \frac{E}{100}\right) \tag{2}$$

When s_f is given by the Jacob approximation for the Theis solution, then B can be found from (Sternberg 1973):

$$s_f = BQ = \frac{Q}{4\pi T}\left[\ln\left(\frac{2.25Tt}{r_w^2 S}\right) + 2s_p\right] \qquad (3)$$

where T = aquifer transmissivity (L^2/T),

S = aquifer storage coefficient (dimensionless),

t = time since pumping began (T),

r_w = effective wellbore radius (L), and

s_p = a partial penetration factor (dimensionless).

In (3), the effect of partial penetration may be represented by (Brons and Marting 1961):

$$s_p = \left(\frac{D-L}{L}\right)\left[\ln\left(\frac{D}{r_w}\right) - G\left\{\frac{L}{D}\right\}\right] \qquad (4)$$

where D = aquifer thickness (L),
L = well screen length (L), and
G = a function of the L/D ratio (dimensionless).

Using available data, Bradbury and Rothschild (1985) expressed G as the polynomial $G = a + b(L/D) + c(L/D)^2 + d(L/D)^3$, where the fitting coefficients were a = 2.948, b = -7.363, c = 11.447, and d = -4.675. Substituting (1) into (3) yields:

$$T = \frac{Q}{4\pi(s_t - s_w)}\left[\ln\left(\frac{2.25Tt}{r_w^2 S}\right) + 2s_p\right] \qquad (5)$$

Well efficiency is embedded in (5) since $s_w = CQ^2$, and C is defined by (2). Hence, a step drawdown test is not required if E can be estimated. In addition, the effect of partial penetration is represented by (4) using the Bradbury-Rothschild polynomial for G. In (5), T appears on both sides of the equation; hence, an iterative solution is required (Bradbury and Rothschild 1985). Initially, a guess is made for T (Tguess in the program) on the right-hand side of (5) and an updated solution for T (Tcalc in the program) is obtained from the left-hand side. This updated solution is again used on the right-hand side of (5) and a new T is again computed. This iterative process continues until some suitable tolerance criterion for error (Err in the program) is reached. For the Matlab program shown in the Appendix, either metric or customary U.S. units may be employed.

Program Usage

The program is executed from the Matlab command line by typing in the m-file program name (i.e., [A, T]=TQs). The user is prompted to select a system of units and then enter input values for Q, s_t, t, L, r_w, S, D (optional), and C (optional). Walton (1970) showed that T is relatively insensitive to variations in *S*; hence this value may be estimated. Tabulated and graphed output consists of a range of *T* values that correspond to a range of expected well efficiencies and aquifer penetration values. The two original examples shown in Bradbury and Rothschild (1985) are used as illustrations. Input data for these tests are summarized in Table 1. The Matlab program is executed once for each test and the user is prompted to enter appropriate data from Table 1. Figure 1 is a graphical representation of the tabulated output for well 1. Output for well 2 was omitted because it is similar to Figure 1. If known values for *D* and *C* are entered, then single best estimates for *T* and *E* are also obtained. Using well 1 metric units from Table 1, we find T = 46.6 m^2/day at E = 99.9% and L/D = 23%; for well 2, T = 36.2 m^2/day at E = 99.9% and L/D = 59%. Bradbury and Rothschild originally reported *T* values of 47.6 and 36.7 m^2/day for wells 1 and 2, respectively. Well efficiencies were determined from (2) using their *C* value.

Parameter (units)	**Well 1 (metric)**	**Well 1 (U.S.)**	**Well 2 (metric)**	**Well 2 (U.S.)**
Q (lpm or gpm)	37.853	10	37.853	10
s_t (m or ft)	4.572	15	2.743	9
t (min)	480	480	480	480
L (m or ft)	14.326	47	20.726	68
r_w (cm or in)	7.62	3	7.62	3
S (dimensionless)	0.0002	0.0002	0.0002	0.0002
D (m or ft)	62.484	205	35.052	115
C (min^2/m^5 or sec^2/ft^5)	3.453	32.7	3.453	32.7

Table 1. Properties for well 1 (metric) were used in the Matlab program to generate Figure 1. A similar figure can be generated with well 2 data.

One may question the choice of having partial penetration as a variable in Figure 1 since a single value for this parameter should be known from the driller's log. However, we often have difficulty actually deciding where aquifer boundaries are located. This is especially true in horizontally stratified aquifers where vertical changes in hydraulic conductivity may not be obvious. In addition, step-drawdown tests that determine *C* are the exception rather than the rule, especially in monitoring well applications. This program simply provides a range of estimated *T* values that can assist us in overcoming these difficulties. As seen above, we can narrow the range of possible *T* values to a single best estimate if we know partial penetration and well efficiency. Alternately, we may determine partial penetration from Figure 1 if we have independent estimates for *T* and *E*. The real value of this exercise, however, may be the recognition of uncertainty in the estimation process.

Conclusions

Specific capacity data are often used in hydrogeological studies to estimate *T*. The major criticism of this method is that it assumes a quasi-steady state condition has been established. This is in contrast to a conventional aquifer test where transient *s* and *t* values are matched to an appropriate theoretical type-curve. However, the Matlab program presented here is really a parameter sensitivity analysis because it translates specific capacity into a range of *T* values that reflect the combined influence of the formation, aquifer penetration, and well efficiency. This type of analysis simply gives us another way to determine *T*. These *T* estimates can be valuable in those situations where conventional aquifer tests are unavailable.

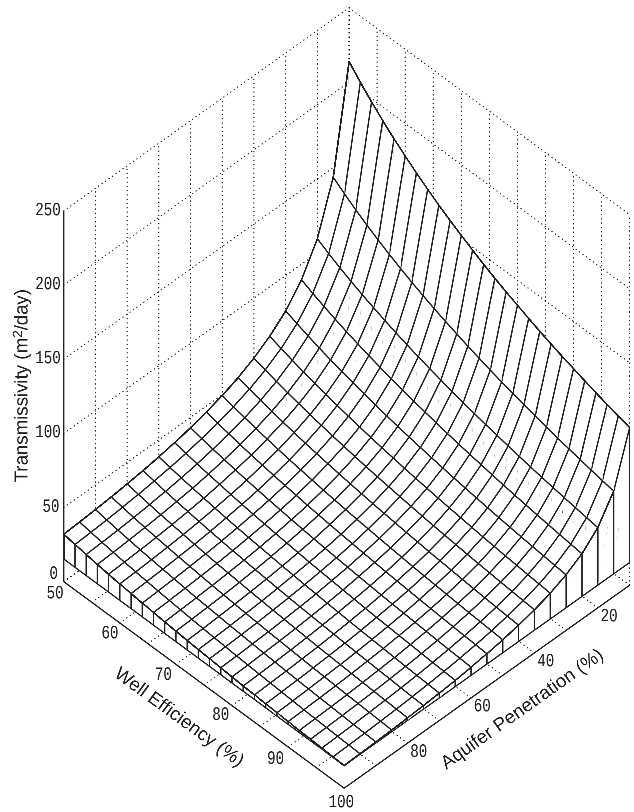

Figure 1. Transmissivity as a function of aquifer penetration and well efficiency for well 1.

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Appendix

```
function [A, T]=TQs
%TQs  computes Transmissivity (T) from Specific Capacity (Q/s) data.
%
%       This m-file was written in the Matlab language by:
%       Stephen G. McLin, 8 May 2003, e-mail: sgm@lanl.gov
%
%       A = a matrix of T values as a function of R and E.
%       Note that R is the last row of A and E is the last column of A
%       T = transmissivity (sq m/day or sq ft/day).
%       Q = well pump rate (lps or gpm).
%       s = wellbore drawdown (m or ft).
%       t = time (minutes).
%       D = aquifer thickness (m or ft).
%       L = well screen length (m or ft).
%       R = L/D (dimensionless penetration).
%       r = wellbore radius (cm or in).
%       S = aquifer storage coefficient (or specific yield).
%       E = well efficiency (%).
%       C = well loss coefficient (min^2/m^5 or sec^2/ft^5).
%
format short;
Units=input(‘Enter 1 for metric units and 2 for US units.......');
if Units==1
   Q=input(‘Enter Q (lpm) now.......'); conv=1000;
   s=input(‘Enter drawdown (m) now.......');
   t=input(‘Enter time (minutes) now.......');
   L=input(‘Enter well screen length (m) now.......');
   r=input(‘Enter wellbore radius (cm) now.......'); r=r/100;
   S=input(‘Enter storage coefficient S now.......');
   Do=input(‘Enter observed aquifer thickness (m) now (enter 1 if unknown).......');
   Co=input(‘Enter step-test C (min^2/m^5) now (enter 1 if unknown).......');
   if Co~=1; Co=Co*3600; end; str='Transmissivity (sq m/day)';
elseif Units==2
   Q=input(‘Enter Q (gpm) now.......'); conv=7.48;
   s=input(‘Enter drawdown (ft) now.......');
   t=input(‘Enter time (minutes) now.......');
   L=input(‘Enter well screen length (ft) now.......');
   r=input(‘Enter wellbore radius (in) now.......'); r=r/12;
   S=input(‘Enter storage coefficient S now.......');
   Do=input(‘Enter observed aquifer thickness (ft) now (enter 1 if unknown).......');
   Co=input(‘Enter step-test C (sec^2/ft^5) now (enter 1 if unknown).......');
   str='Transmissivity (sq ft/day)';
else
   error('You have entered an incorrect response.   Please start again.');
end
```

```
E=[50:2:100]'; [n1,m1]=size(E);
R=[0.1:0.05:1.0]'; [n2,m2]=size(R); D=L./R;
A=zeros(n1+1,n2+1); err=0.000001; Tguess=1.0;
a=2.948; b=-7.363; c=11.447; d=-4.675;
C=(1-E./100).*(s/Q^2); sw=C.*Q^2;
G=(a+b*(L./D)+c*(L./D).^2+d*(L./D).^3);
sp=((D-L)./L.*(log(D./r)-G));
for j=1:n2; for i=1:n1;
   Tcalc(i,j)=1440*Q*(log(2.25*Tguess*t/(1440*r^2*S))+2*sp(j))/(4*conv*pi*(s-sw(i)));
   diff=abs(Tcalc(i,j)-Tguess); test=diff;
   while test>err
      Tcalc(i,j)=1440*Q*(log(2.25*Tguess*t/(1440*r^2*S))+2*sp(j))/(4*conv*pi*(s-sw(i)));
      diff=abs(Tcalc(i,j)-Tguess); Tguess=Tcalc(i,j); test=diff;
   end; A(i,j)=Tcalc(i,j);
end; end
A(1:n1,(n2+1))=E; A((n1+1),1:n2)=100.*R';
z=A(1:n1,1:n2); x=100.*R; y=E; h=figure;
set(h,'PaperPosition',[0.25,0.25,8.00,10.50]);
meshz(x,y,z); zlabel(str);
ylabel('Well Efficiency (%)'); xlabel('Aquifer Penetration (%)');
if Do==1; T=1; return;
elseif Co==1; T=1; return;
else
fac=60*60*conv*conv;
Eo=100*(1-Co*Q^2/(s*fac)); swo=Co*Q^2/fac;
Go=a+b*(L/Do)+c*(L/Do)^2+d*(L/Do)^3;
spo=(Do-L)/L*(log(Do/r)-Go);
Tcalco=1440*Q*(log(2.25*Tguess*t/(1440*r^2*S))+2*spo)/(4*conv*pi*(s-swo));
   diff=abs(Tcalco-Tguess); test=diff;
   while test>err
      Tcalco=1440*Q*(log(2.25*Tguess*t/(1440*r^2*S))+2*spo)/(4*conv*pi*(s-swo));
      diff=abs(Tcalc-Tguess); Tguess=Tcalco; test=diff;
   end; T=[Tcalco Eo L*100/Do]; end;
% Tcalco = best single estimate for transmissivity;
% Eo = well efficiency; 100L/Do = aquifer penetration;
```